Answer on Question #42067 -Math -Statistics and Probability

In a case of 144 boxes of light bulbs, it is found that exactly 4 boxes contain at least one broken light bulb. If 5 boxes are selected from the case at random, what is the probability (rounded to 4 decimal places) that none of the boxes will contain a broken light bulb?

Solution.

For a start, we number the boxes. Let, boxes with numbers from 1 to 4 contain at least one broken light bulb and boxes with number from 5 to 144 don't contain broken bulbs.

Let $\Omega = \{(\omega_1, \omega_2, \dots, \omega_5) |\ \omega_i = \overline{1, 144}, \omega_i \neq \omega_j, if\ i \neq j\}-$ probability space. $|\Omega| = C_{144}^5$, where $|\Omega|$ is a power of the probability space.

Now, we find the probability that none of the boxes will contain a broken light bulb. We denote it $P(A)$.

$A = \{(\omega_1, \omega_2, \dots, \omega_5) \in \Omega | \omega_i = \overline{5, 144}, \omega_i \neq \omega_j, if\ i \neq j\}, |A| = C_{140}^5$, where $|A|$ is a power of A.

Using classical definition of probability, we have

$$P(A) = \frac{|A|}{|\Omega|} = \frac{C_{140}^5}{C_{144}^5} = \frac{140!}{5! \cdot 135!} \cdot \frac{5! \cdot 139!}{144!} = \frac{136 \cdot 137 \cdot 138 \cdot 139}{141 \cdot 142 \cdot 143 \cdot 144} = 0.8669$$

Answer: the probability (rounded to 4 decimal places) that none of the boxes will contain a broken light bulb is 0.8669.