A civil engineer is analyzing the compressive strength of concrete. A random sample of 12 specimens has a mean compression strength of 3250 psi with variance 1000 (psi)2.

i)   Construct a 95% confidence interval for the population mean compressive strength.
ii) Construct a 95% confidence interval for the population standard deviation of compressive strength

Solution

$n = 12, \ \bar{x} = 3250, s = 1000.$

i)

A 95% confidence interval for the population mean compressive strength is

$$\left(\bar{x} - t_{n-1}\frac{s}{\sqrt{n}}; \bar{x} + t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}}\right) = \left(3250 - t_{11}\frac{1000}{\sqrt{12}}; 3250 + t_{11}\frac{1000}{\sqrt{12}}\right).$$

It is $\left(3250 - t_{11}\frac{1000}{\sqrt{12}}; 3250 + t_{11}\frac{1000}{\sqrt{12}}\right) =$

$$= (3250 - TINV(0,05; 11) * 1000/SQRT(12); 3250 + TINV(0,05; 11) * 1000/SQRT(12))$$

$$= \left(3250 - 2.2 * \frac{1000}{SQRT(12)}; 3250 + 2.2 * \frac{1000}{SQRT(12)}\right) =$$

$$= \left(3250 - 2.2 * \frac{1000}{3.464}; 3250 + 2.2 * \frac{1000}{3.464}\right) = (2614.630; 3885.370)$$

using Excel.

Note that Excel gives a two-tailed value for t-value, i.e., we do not divide α in half.

ii)

A 95% confidence interval for the population standard variance of compressive strength is

$$\left(\sqrt{\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2},n-1}}}; \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}}\right) = \left(\sqrt{\frac{(12-1)1000^2}{\chi^2_{0.025,11}}}; \sqrt{\frac{(12-1)1000^2}{\chi^2_{0.975,11}}}\right) = \left(\sqrt{\frac{(12-1)1000^2}{21.92}}; \sqrt{\frac{(12-1)1000^2}{3.81}}\right).$$

It is

$$\left(\sqrt{\frac{(12-1)1000^2}{\chi^2_{0.025,11}}}; \sqrt{\frac{(12-1)1000^2}{\chi^2_{0.975,11}}}\right)$$

$$= \left(1000 * SQRT\left(\frac{12-1}{CHIINV(0,025; 11)}\right); 1000\right.$$

$$\left. * SQRT\left(\frac{12-1}{CHIINV(0,975; 11)}\right)\right) =$$

$$= \left(1000 * \sqrt{\frac{12-1}{21.92}}; 1000 * \sqrt{\frac{12-1}{3.816}}\right) = (708.395; 1697.878)$$

using Excel.

Answer: i) (2614.630; 3885.370); **ii)** (708.395; 1697.878).