

Answer on Question #42032 – Math – Calculus

What is the common property between determinants, integrals and derivatives?

Solution.

All these operations are linear, i.e. the property

$$L(\alpha a + \beta b) = \alpha \cdot La + \beta \cdot Lb$$

obeys for all objects a and b of common nature (among three objects, that are named) and all the numbers α and β .

But we must determine what this operation means for determinants.

If we write an $n \times n$ matrix in terms of its column vectors $A = [a_1, a_2, \dots, a_n]$, where the a_j are vectors of size n , then the determinant of A is defined so that

$$|a_1, \dots, ba_j + cv, \dots, a_n| = b \cdot |A| + c \cdot |a_1, \dots, v, \dots, a_n|,$$

where b and c are scalars, v is any vector of size n .

This equation says that the determinant is a linear function of each column.

A similar procedure is determined for strings of a determinant.

Note, that this property can not be right, if an integral becomes divergent.

Answer: linearity.