Answer on Question #41959 – Math – Other:

Consider a purely probabilistic game that you have the opportunity to play. Each time you play there are n potential outcomes $x_1, x_2, \ldots, x_n$ (each of which is a specified gain or loss of euros). These outcomes $x_1, x_2, \ldots, x_n$ occur with the probabilities $p_1, p_2, \ldots, p_n$ respectively (where $p_1 + p_2 + \ldots + p_n = 1$ and $\forall\, 1 \le i \le n{:}\, p_i \in [0,1]$). Positive xi values mean a gain of $|x_i|$ euros and negative values mean a loss of $|x_i|$ euros. Assume that $x_1, x_2, \ldots, x_n$ and $p_1, p_2, \ldots, p_n$ are all known quantities. Furthermore, assume that each play of the game takes up one hour of your time, and that only you can play the game (you can't hire someone to play for you).
Let M be the game's expected value. That is, $M = p_1 * x_1 + p_2 * x_2 + \ldots + p_n * x_n$. Let S be the game's standard deviation. That is:

$$S = \sqrt{p_1 * (x_1 - M)^2 + p_2 * (x_2 - M)^2 + \ldots + p_n * (x_n - M)^2};$$

In the real world, should a rational player always play this game whenever the expected value M is not negative? Yes/NO. Explain

Solution.

Consider the following case:

$$n = 4, p_1 = p_2 = p_3 = 0.33, p_4 = 0.01, x_1 = x_2 = x_3 = -10, x_4 = 1000;$$

Hence:

$$M = 0.01 \cdot 1000 - 3 \cdot 0.33 \cdot 10 = 10 - 9.9 = 0.1 > 0;$$

$$S = \sqrt{0.01 \cdot 999.9^2 + 0.99 \cdot 10.1^2} = \sqrt{10098.99} \approx 100;$$

So the expected value is positive. But if one plays this game, he will lose 10 euros with the probability 99%.

In this situation one can win after n games with the probability

$$P = 1 - 0.99^n;$$

Assume that rational player won't play more than 8 hours. So, he will be in the black with the probability

$$1 - 0.99^8 \approx 0.08;$$

So we conclude that a rational player should not play this game.