

Answer on Question #41943, Math, Statistics and Probability

The lifetime X of a bulb is a random variable with the probability density function:

$$f(x) = \begin{cases} 6 \cdot [0.25 - (x - 1.5)^2], & \text{when } 1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

X is measured in multiples of 1000 hrs. What is the probability that none of the three bulbs in a traffic signal have to be replaced in the first 1500 hrs of their operation?

Solution

Let T_i is the random variable of a lifetime (is measured in multiples of 1000 hrs) of i -th bulb, where $i = 1, 2, 3$. If $A =$ "lifetime of each bulb is longer than 1500 hrs", then

$$P(A) = P(T_1 \geq 1.5) \cdot P(T_2 \geq 1.5) \cdot P(T_3 \geq 1.5).$$

$$\begin{aligned} P(T_1 \geq 1.5) &= \int_{1.5}^{\infty} f(x) dx = \int_{1.5}^2 6 \cdot [0.25 - (x - 1.5)^2] dx = \left(\frac{3}{2}x - 2(x - 1.5)^3 \right) \Big|_{1.5}^2 \\ &= (3 - 2 \cdot 0.5^3) - \frac{9}{4} = \frac{1}{2}. \end{aligned}$$

So

$$P(A) = \left(\frac{1}{2} \right)^3 = \frac{1}{8}.$$

Answer: $\frac{1}{8}$.