Answer on Question #41937, Math, Statistics and Probability

A random sample of 10 mobile phone batteries has a lifetime with variance 16 months. Assuming the lifetime of batteries to be normally distributed, construct a 95% confidence interval for the variance of all such mobile phone batteries.

Solution

For a confidence level $1 - \alpha$ confidence interval for the variance is

$$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}},$$

where s^2 is a sample variance, n is a sample size.

For a sample size of $n = 10$, we will have $df = n - 1 = 9$ degrees of freedom. For a 95% confidence interval, we have $\alpha = 0.05$, which gives 2.5% of the area at each end of the chi-square distribution. We find values of $\chi^2_{0.975} = 2.700$ and $\chi^2_{0.025} = 19.023$. This leads to the inequality for the variance

$$\frac{9 \cdot 16}{19.023} \leq \sigma^2 \leq \frac{9 \cdot 16}{2.700} \rightarrow 7.570 \leq \sigma^2 \leq 53.333.$$

Answer: $7.570 \leq \sigma^2 \leq 53.333$.