Answer on Question #41869 – Math – Statistics and Probability

The sample mean is $\bar{x} = \frac{63+63+66+67+68+69+70+70+71+71}{10} = \frac{678}{10} = 67.8$ (or via an Excel function =AVERAGE(63; 63; 66; 67; 68; 69; 70; 70; 71; 71))

The sample standard deviation is

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}} =$$

$$= \sqrt{\frac{2(63-67.8)^2+(66-67.8)^2+(67-67.8)^2+(68-67.8)^2+(69-67.8)^2+2(70-67.8)^2+2(71-67.8)^2}{9}} = 3.01$$ (or via an Excel function =STDEV(63; 63; 66; 67; 68; 69; 70; 70;71; 71)).

The formulation of the null and alternative hypotheses should be

$H_0: \mu = 66$ versus $H_1: \mu \neq 66$.

The *t* test statistic is

$T = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}} = \frac{67.8-66}{\frac{3.01}{\sqrt{10}}}$=1.89, degrees of freedom d.f.=9.

Method 1

We test at the level of significance $\alpha = 0.05$. Since H_1 is two-tailed, we set the rejection region

$R: |T| \geq t_{0.025}$.

From the *t* table we find that $t_{0.025}$ with d.f.=9 is 2.262. Because the observed value t=1.89 is smaller than 2.262, the null hypothesis is not rejected at $\alpha = 0.05$.

Conclusion: there is strong evidence that the mean weight in population is 66 kg (with $\alpha = 0.05$).

Method 2

In two-tailed test p-value is the sum of area in two tails, so

$p - value = P(t \leq -t_{calculated}) + P(t \geq t_{calculated}) = P(t \leq -1.89) + P(t \geq 1.89) =$

=2 $P(t \geq 1.89) = 0.09 > 0.05 = \alpha$. To find it, we calculate via excel the function TDIST(1,89;9;2) or seek from t tables with 9 degrees of freedom $t_{0.05} = 1.833$ and $t_{0.025} = 2.262$, therefore the p-value is between 2*0.05=0.1 and 2*0.025=0.05, in any case it is greater than 0.05. We do not reject the null hypothesis, because p-value is greater than $\alpha = 0.05$.

Conclusion: there is strong evidence that the mean weight in population is 66 kg (with $\alpha = 0.05$).

Method 3

A 95% confidence interval for μ is

$$\left(\bar{x} - t_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}; \bar{x} + t_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}\right) = \left(67.8 - 2.262 * \frac{3.01}{\sqrt{10}}; 67.8 + 2.262 * \frac{3.01}{\sqrt{10}}\right) = (65.65; 69.95)$$

We can see $\mu_0 = 66 \textit{ lies within the } 95\%$ confidence interval. The null hypothesis will not be rejected at level $\alpha = 0.05$ if μ_0 lies within the 95% confidence interval.

Conclusion: there is strong evidence that the mean weight in population is 66 kg (with $\alpha = 0.05$).