Answer on Question # 41846, Math, Statistics and Probability

100 people are asked to choose a name for a baby. They are given 14 names to choose from. What is the probability 2/3 of the 100 people will choose the same name?

Solution.

There are 14^{100} ways to 100 people to choose a name for a baby from 14 given names.
There are $14 \cdot \binom{100}{67} \cdot 13^{100-67} = 14 \cdot \binom{100}{67} \cdot 13^{33}$ ways to exactly 67 people to choose the same name for a baby from 14 given names.
There are $14 \cdot \binom{100}{68} \cdot 13^{100-68} = 14 \cdot \binom{100}{68} \cdot 13^{32}$ ways to exactly 68 people to choose the same name for a baby from 14 given names.
…
There are $14 \cdot \binom{100}{100} \cdot 13^{100-100} = 14$ ways to exactly 100 people to choose the same name for a baby from 14 given names.
Since $\left\lfloor \frac{2}{3} \cdot 100 \right\rfloor = 67$, the probability equals

$$\frac{14 \cdot \binom{100}{67} \cdot 13^{33} + 14 \cdot \binom{100}{68} \cdot 13^{32} + \cdots + 14}{14^{100}}$$

Answer:

$$\frac{14 \cdot \binom{100}{67} \cdot 13^{33} + 14 \cdot \binom{100}{68} \cdot 13^{32} + \cdots + 14}{14^{100}} = \frac{\sum_{i=1}^{34} \frac{14 * 100! * 13^{34-i}}{(34-i)!\,(i+66)!}}{14^{100}} =$$

$$= \frac{24667293929634099269021390894907631390998566409683268608861906362}{14^{100}}$$

$$\approx 6.02 * 10^{-51}$$