

### Answer on Question #41815 – Math – Complex Analysis

Let function  $f(z)$  be analytic in  $0 < |z - z_0| < R$ . Prove that integrals of  $f$  are path independent in  $0 < |z - z_0| < R$  if and only if  $\text{Res}_{z_0} f = 0$

#### *Proof*

If a function  $f$  is analytic in the region  $0 < |z - z_0| < R$ , then according to the main theorem of residues theory, the integral

$$\int_{\gamma^+} f(z) dz = 2\pi i \cdot \text{res}_{z_0} f(z),$$

where the direction of the path  $\gamma$  is counterclockwise.

If case of the opposite direction of the path  $\gamma$ ,

$$\int_{\gamma^-} f(z) dz = -2\pi i \cdot \text{res}_{z_0} f(z).$$

So, the integral  $\int_{\gamma} f(z) dz$  is path independent for all the paths in the region  $0 < |z - z_0| < R$ , if and only if  $\text{res}_{z_0} f(z) = 0$ .