Answer on Question#41801, Math, Other

The heights of a group of 1000 people are measured, and found to be distributed normally with a mean height of 178 cm and a standard deviation of 12 cm.
(a) How many people do we expect to find with a height of 178 cm or less?
(b) How many people do we expect to find with a height between 166 cm and 190 cm?

In probability theory, the normal (or Gaussian) distribution is a very commonly occurring continuous probability distribution—a function that tells the probability that any real observation will fall between any two real limits or real numbers, as the curve approaches zero on either side. Normal distributions are extremely important in statistics and are often used in the natural and social sciences for real-valued random variableswhose distributions are not known.

The normal distribution is immensely useful because of the central limit theorem, which states that, under mild conditions, the mean of many random variables independently drawn from the same distribution is distributed approximately normally, irrespective of the form of the original distribution: physical quantities that are expected to be the sum of many independent processes (such as measurement errors) often have a distribution very close to the normal. Moreover, many results and methods (such as propagation of uncertainty and least squares parameter fitting) can be derived analytically in explicit form when the relevant variables are normally distributed.

The Gaussian distribution is sometimes informally called the bell curve. However, many other distributions are bell-shaped (such as Cauchy's, Student's, and logistic). The terms Gaussian function and Gaussian bell curve are also ambiguous because they sometimes refer to multiples of the normal distribution that cannot be directly interpreted in terms of probabilities.

A normal distribution is

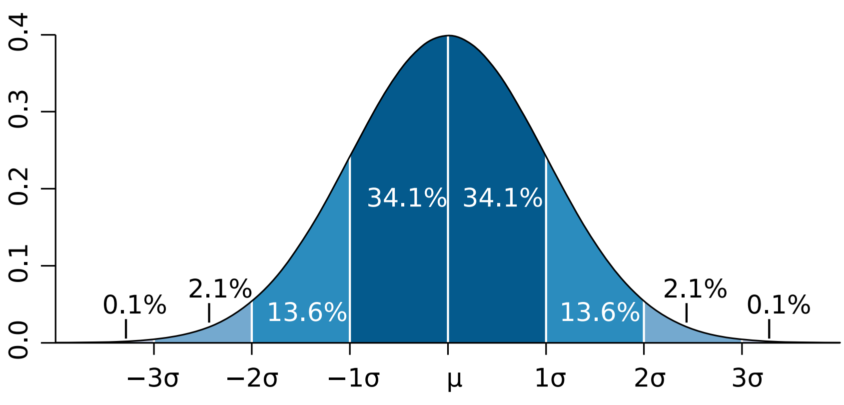
$$f(x,\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

The parameter μ in this definition is the mean or expectation of the distribution (and also its median andmode). The parameter σ is its standard deviation; its variance is therefore σ 2. A random variable with a Gaussian distribution is said to be normally distributed and is called a normal deviate.

If μ = 0 and σ = 1, the distribution is called the standard normal distribution or the unit normal distribution, and a random variable with that distribution is a standard normal deviate.

Answers:

a)



From the plot above you can see, that the normal distribution is symmetric.
How many people do we expect to find with a height of 178 cm or less? - ANSWER is 500.
To find an answer we have to integrate from $-\infty$ to 178 cm.

$$N=1000\int_{-\infty}^{178}\frac{1}{12\sqrt{2\pi}}e^{\frac{-(x-178)^2}{2*12^2}}dx=½*1000=500$$

(b) How many people do we expect to find with a height between 166 cm and 190 cm?

We can notice that 166 = 178-12, and 190=178+12. So [166, 190] cm is 1 σ region.
From the plot above we can notice that in 1 σ we can find 68.2 %.

So, N = 1000*0.682 = 682.
We can calculate it in other way:

$$N=1000\int_{166}^{190}\frac{1}{12\sqrt{2\pi}}e^{\frac{-(x-178)^2}{2*12^2}}dx=0.682*1000=682$$

FINAL RESULTS:
a) N = 500
b) N=682