Answer on Question #41684 – Math – Analytic Geometry

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} uvcos(\alpha) \\ uvsin(\alpha) \\ \frac{1}{2}(u^2 - v^2) \end{pmatrix}$$

Derivatives of the radius vector:

$$\overrightarrow{r_u} = \begin{pmatrix} x_u \\ y_u \\ z_u \end{pmatrix} = \begin{pmatrix} vcos(\alpha) \\ vsin(\alpha) \\ u \end{pmatrix}; \overrightarrow{r_v} = \begin{pmatrix} x_v \\ y_v \\ z_v \end{pmatrix} = \begin{pmatrix} ucos(\alpha) \\ usin(\alpha) \\ -v \end{pmatrix}; \overrightarrow{r_\alpha} = \begin{pmatrix} x_\alpha \\ y_\alpha \\ z_\alpha \end{pmatrix} = \begin{pmatrix} -uvsin(\alpha) \\ uvcos(\alpha) \\ 0 \end{pmatrix};$$

Scalar products:

$$\overrightarrow{r_u} \cdot \overrightarrow{r_v} = uvcos^2(\alpha) + uvsin^2(\alpha) - uv = 0$$

$$\overrightarrow{r_u} \cdot \overrightarrow{r_\propto} = -uv^2cos(\alpha)sin(\alpha) + uv^2cos(\alpha)sin(\alpha) = 0$$

$$\overrightarrow{r_v} \cdot \overrightarrow{r_\propto} = -u^2vcos(\alpha)sin(\alpha) + u^2vcos(\alpha)sin(\alpha) = 0$$

It means that $\overrightarrow{r_u}, \overrightarrow{r_v}$ and $\overrightarrow{r_\alpha}$ can be chosen as a basis and these vectors are orthogonal.