Find the direction in which the function $f = x^2 - y^2 + 2xy$ decreases more rapidly at the point (1,1).

Solution.

Partial derivatives of the function: $\dfrac{\partial f}{\partial x} = 2x + 2y$, $\dfrac{\partial f}{\partial y} = -2y + 2x$.

The values of the partial derivatives of the function at the point $A(1;1)$:

$\left.\dfrac{\partial f}{\partial x}\right|_A = 2 + 2 = 4$, $\left.\dfrac{\partial f}{\partial y}\right|_A = -2 + 2 = 0$.

The value of the gradient of the function in the direction of the unit vector $\vec{n}(n_x; n_y)$ at the point $A(1;1)$:

$$\left.\frac{\partial f}{\partial x}\right|_A \cdot n_x + \left.\frac{\partial f}{\partial y}\right|_A \cdot n_y = 4 \cdot n_x + 0 \cdot n_y = 4n_x. \tag{1}$$

As $n_x^2 + n_y^2 = 1$ for a unit vector, then the minimal value of (1) reaches if $n_x = -1$.

So, the function decreases more rapidly at the given point at the negative direction of *X*-axis.

Answer: in the direction of the vector $(-1; 0)$.