

Answer on Question #4157 – Math – Trigonometry

how many solution sets of $\cos^2 \theta$ are possible?

Solution:

Initial equation:

$$\cos^2 \theta = \alpha$$

For $0 \leq \alpha \leq 1$:

$$\cos^2 \theta = \alpha \Rightarrow$$

$$1) \cos \theta = \sqrt{\alpha}; \quad 2) \cos \theta = -\sqrt{\alpha}$$

First case:

$$\cos \theta = \sqrt{\alpha}$$

$$\theta = \pm \arccos(\sqrt{\alpha}) \pm 2\pi n, n \in \mathbb{Z}$$

Second case:

$$\cos \theta = -\sqrt{\alpha}$$

$$\theta = \pm \arccos(-\sqrt{\alpha}) \pm 2\pi n, n \in \mathbb{Z}$$

Hence, for $0 \leq \alpha \leq 1$ we have has infinitely many solutions.

$\alpha \in (-\infty, 0) \cup (1, +\infty)$:

$$-1 \leq \cos \theta \leq 1 \Rightarrow 0 \leq \cos^2 \theta \leq 1$$

Thus, for values $\alpha > 1$ and $\alpha < 0$ an equation has no solution.

Answer: number of solutions: $\begin{cases} \infty & \text{for } 0 \leq \alpha \leq 1 \\ 0 & \text{for } \alpha \in (-\infty, 0) \cup (1, +\infty) \end{cases}$