

Answer on question 41395 – Math – Statistics and Probability

A company produces one-kilogram sugar packets. The specifications on the net content are 1000 ± 5 grams. Assuming that the net content follows normal distribution with mean weight as 1005 grams and the process capability equal to 30 grams, find out the proportion of packets that have weight less than lower specification limit. What should be the mean if this proportion is to be reduced to 0.01?

Solution

First let us find the standard deviation of this process. It is known that

$$C_p(\text{process capability}) = \frac{C_{pu} + C_{pl}}{2} = \frac{1}{2} \left(\frac{USL - m}{3\sigma} + \frac{m - LSL}{3\sigma} \right) = \frac{1005 - 995}{6\sigma} = 30$$

Therefrom

$$\sigma = \frac{1005 - 995}{6 * 30} = \frac{1}{18}$$

We know that the process has a normal distribution so get

$$CDF(x = 995) = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{x - m}{\sqrt{2}\sigma} \right) \right] \approx 0.75.$$

To find the mean if this proportion is to be reduced to 0.01 we should solve the following

$$\frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{x - m}{\sqrt{2}\sigma} \right) \right] = 0.01$$

$$\operatorname{erf} \left(\frac{x - m}{\sqrt{2}\sigma} \right) = -0.98$$

$$\frac{995 - m}{\sqrt{2} * \frac{1}{18}} = -1.64$$

$$m = 995.13.$$

Answer: 0.75; 995.13.