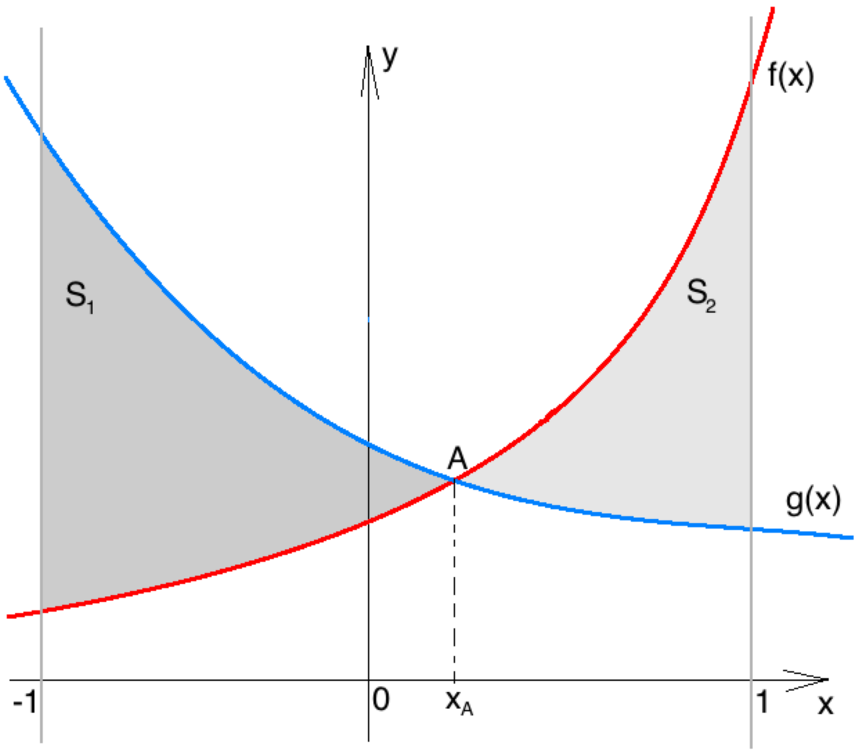
Answer on Question #41478 – Math - Calculus

Sketch the region bounded by f(x) = 1+2ex, g(x) = 1+4e−x, x = −1 and x = 1. Using calculus, find the area of the region, showing all the working. Express your answer in simplified exact form.

step by step working out

Solution:



$$f(x) = 1 + 2e^{x}$$
$$g(x) = 1 + 4e^{-x}$$

Area of the region can be represented as the sum of two areas:

$$S = S_1 + S_2 \qquad (1)$$

First, we must determine the x-intercept (point A):

$$f(x) = g(x)$$
$$1 + 2e^{x} = 1 + 4e^{-x}$$
$$e^{x} = \frac{2}{e^{x}}$$
$$e^{2x} = 2$$
$$2x = \ln(2)$$
$$x = \frac{\ln(2)}{2} = \ln(\sqrt{2}) = x_A$$

Now we can find area S_1 and S_2:

Formula for finding the area between two curves:

$$\text{Area} = \int_a^b (f_1(x) - f_2(x))dx$$

Area S_1: (boundaries are $x = -1, x = x_A = \ln(\sqrt{2})$)

$$S_1 = \int_{-1}^{\ln(\sqrt{2})} (g(x) - f(x))dx = \int_{-1}^{\ln(\sqrt{2})} (1 + 4e^{-x} - (1 + 2e^x))dx =$$

$$= 2\int_{-1}^{\ln(\sqrt{2})} (2e^{-x} - e^x)dx = 2(-2e^{-x} - e^x)\Big|_{-1}^{\ln(\sqrt{2})} =$$

$$= 2\left(-\sqrt{2} + 2e - \sqrt{2} + \frac{1}{e}\right) = 2\left(-2\sqrt{2} + 2e + \frac{1}{e}\right)$$

Area S_2: (boundaries are $x = x_A = \ln(\sqrt{2}), x = 1$)

$$S_1 = \int_{\ln(\sqrt{2})}^{1} (f(x) - g(x))dx = \int_{\ln(\sqrt{2})}^{1} ((1 + 2e^x) - (1 + 4e^{-x}))dx =$$

$$= 2\int_{\ln(\sqrt{2})}^{1} (e^x - 2e^{-x})dx = 2(e^x - 2e^{-x})\Big|_{\ln(\sqrt{2})}^{1} =$$

$$= 2\left(e - \sqrt{2} + \frac{2}{e} - \sqrt{2}\right) = 2\left(-2\sqrt{2} + e + \frac{2}{e}\right) \Longrightarrow (1)$$

$$S = S_1 + S_2 = 2\left(-2\sqrt{2} + 2e + \frac{1}{e}\right) + 2\left(-2\sqrt{2} + e + \frac{2}{e}\right) = -8\sqrt{2} + \frac{6}{e} + 6e$$

Answer: $S = -8\sqrt{2} + \frac{6}{e} + 6e$.