Answer on Question # 41476 – Math – Calculus

Let f(x) = 8+(2x+3)ln(√x+x−1)
(a) Find the derivative of f.
(b) Using the derivative, estimate f(1.1).
need step by step working out

Solution:

(a) $f(x) = 8 + (2x + 3)\ln(\sqrt{x} + x - 1)$

Derivative of f(x):

$$\frac{df(x)}{dx} = \frac{d\left(8 + (2x + 3)\ln(\sqrt{x} + x - 1)\right)}{dx} =$$

$$= |\text{derivative of a constant is zero, } (8)' = 0| =$$

$$= \frac{d\left((2x + 3)\ln(\sqrt{x} + x - 1)\right)}{dx}$$

$$= |\text{derivative of the multiplication of two function: } (a \cdot b)' = a' \cdot b + b' \cdot a| =$$

$$= \frac{d(2x + 3)}{dx}\ln(\sqrt{x} + x - 1) + (2x + 3)\frac{d\left(\ln(\sqrt{x} + x - 1)\right)}{dx} =$$

$$= |\text{derivative of } (2x + 3)' = 2 + 0 = 2| =$$

$$= 2\ln(\sqrt{x} + x - 1) + (2x + 3)\frac{d\left(\ln(\sqrt{x} + x - 1)\right)}{dx} =$$

$$= \left|\text{derivative of } \left(\ln(\sqrt{x} + x - 1)\right)' = \frac{\frac{d(\sqrt{x} + x - 1)}{dx}}{\sqrt{x} + x - 1}\right| =$$

$$= 2\ln(\sqrt{x} + x - 1) + \frac{(2x + 3)}{\sqrt{x} + x - 1}\frac{d(\sqrt{x} + x - 1)}{dx} =$$

$$= \left|\text{derivative of } (\sqrt{x} + x - 1)' = \frac{1}{2\sqrt{x}} + 1\right| =$$

$$= 2\ln(\sqrt{x} + x - 1) + \frac{(2x + 3)\left(\frac{1}{2\sqrt{x}} + 1\right)}{\sqrt{x} + x - 1}.$$

(b)

$$f(a)' \approx \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

$$f(a)'\Delta x = f(a + \Delta x) - f(a)$$

$$f(a + \Delta x) = f(a)'\Delta x - f(a)$$

In our case we can easily find $f(1)$ *and* $f'(1)$: $a + \Delta x = 1.1 \Longrightarrow a = 1; \quad \Delta x = 0.1$

$$f(1.1) = f(1)' \cdot 0.1 - f(1) = 0.1 \cdot 2\ln(\sqrt{1} + 1 - 1) + \frac{(2 + 3)\left(\frac{1}{2\sqrt{1}} + 1\right)}{\sqrt{1} + 1 - 1} -$$

$$-\left(8 + (2 + 3)\ln(\sqrt{1} + 1 - 1)\right) = 7.5 - 8 = -0.5$$

Answer: (a) $f(x) = 2\ln(\sqrt{x} + x - 1) + \frac{(2x+3)\left(\frac{1}{2\sqrt{x}}+1\right)}{\sqrt{x}+x-1}$ (b) $f(1.1) = -0.5$.