A medical doctor wishes to test the claim that the standard deviation of the systolic blood pressure of deep sea divers is less than 450. To do so, she selected a random sample of 20 divers and found s = 432.
Assuming that the systolic blood pressures of deep sea divers are normally distributed, if the doctor wanted to test her research hypothesis at the .01 level of significance, what is the critical value?

Solution

Let $\sigma_0^2 = 450^2, n = 20, s = 432, \alpha = .01$
One-tailed test:

$$H_0: \sigma^2 = \sigma_0^2$$
$$H_0: \sigma^2 < \sigma_0^2$$

Test statistics

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Rejection region:
Reject H_0 if
$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} < \chi_{1-\alpha;n-1}^2$, where $P(\chi^2 > \chi_{1-\alpha;n-1}^2) = 1 - \alpha$.
Critical value $\chi_{1-\alpha;n-1}^2 = \chi_{1-\alpha;n-1}^2 = \chi_{.99;19}^2$=7.63273.
Test statistics

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{19 * 432^2}{450^2} = 17.5104$$

Conclusion
We do not have enough evidence to reject H_0 at $\alpha = 0.01$
Answer: critical value is 7.63273.