

Answer on Question#41427 – Math - Calculus

Task:

The diameter of a cat's pupil is given by $f(x) = \frac{160x^{-0.4} + 90}{4x^{-0.4} + 15}$ where x is the intensity of light on the pupils.

1. By considering \lim as x tends to infinity of $f(x)$, determine the diameter of the cat's pupils due to very intense light.
2. Show that $f(x)$ may be written as $f(x) = \frac{160+90x^{0.4}}{4+15x^{0.4}}$
3. Deduce the diameter of the cat's pupil for as light diminishes to a minimum intensity.

Solution:

$$1. \lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{160x^{-0.4} + 90}{4x^{-0.4} + 15} = \lim_{x \rightarrow \infty} \frac{\frac{160}{x^{0.4}} + 90}{\frac{4}{x^{0.4}} + 15} = \frac{90}{15} = 6.$$

$$2. f(x) = \frac{160x^{-0.4} + 90}{4x^{-0.4} + 15} = \frac{\frac{160}{x^{0.4}} + 90}{\frac{4}{x^{0.4}} + 15} * 1 = \frac{\frac{160}{x^{0.4}} + 90}{\frac{4}{x^{0.4}} + 15} * \frac{x^{0.4}}{x^{0.4}} = \frac{\left(\frac{160}{x^{0.4}} + 90\right) * x^{0.4}}{\left(\frac{4}{x^{0.4}} + 15\right) * x^{0.4}} = \frac{160 + 90x^{0.4}}{4 + 15x^{0.4}}.$$

$$3. \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{160x^{-0.4} + 90}{4x^{-0.4} + 15} = \lim_{x \rightarrow 0} \frac{160 + 90x^{0.4}}{4 + 15x^{0.4}} = \frac{160}{4} = 40.$$

Answer:

So, the diameter of the cat's pupils due to very intense light is 6. And the diameter of the cat's pupil for as light diminishes to a minimum intensity is 40.