Answer on Question # 41422 – Math - Other

if a b c d are in continued proportion then(ma^3+nb^3-rc^3):(mb^3+nc^3-rd^3)=

Solution.

If a, b, c, d are in continued proportion, then

$$\frac{a}{b}=\frac{b}{c}=\frac{c}{d}\,; \qquad \frac{a}{b}=\frac{c}{d} \;\Rightarrow\; \frac{d}{a}=\frac{bc}{a^2},\;\; \frac{d}{b}=\frac{c}{a}$$

Consider our fraction:

$$\frac{ma^3+nb^3-rc^3}{mb^3+nc^3-rd^3}=\frac{ma^3}{mb^3+nc^3-rd^3}+\frac{nb^3}{mb^3+nc^3-rd^3}-\frac{rc^3}{mb^3+nc^3-rd^3}$$

Consider the first term:

$$\frac{ma^3}{mb^3+nc^3-rd^3}=\frac{m}{m\frac{b^3}{a^3}+n\frac{c^3}{a^3}-r\frac{d^3}{a^3}}=\frac{m}{m\frac{b^3}{a^3}+n\frac{c^3}{a^3}-r\frac{b^3c^3}{a^6}}$$

The second term:

$$\frac{nb^3}{mb^3+nc^3-rd^3}=\frac{n}{m+n\frac{c^3}{b^3}-r\frac{d^3}{b^3}}=\frac{n}{m+n\frac{b^3}{a^3}-r\frac{c^3}{a^3}}$$

The third term:

$$\frac{rc^3}{mb^3+nc^3-rd^3}=\frac{r}{m\frac{b^3}{c^3}+n\;-r\frac{d^3}{c^3}}=\frac{r}{m\frac{a^3}{b^3}+n\;-r\frac{b^3}{a^3}}$$

Let $\frac{b^3}{a^3}=x, \frac{c^3}{a^3}=y$. Then $\frac{y}{x}=\frac{c^3}{b^3}=\frac{b^3}{a^3}=x \;\Rightarrow\; y=x^2$:

$$\frac{ma^3+nb^3-rc^3}{mb^3+nc^3-rd^3}=\frac{m}{mx+ny-rxy}+\frac{n}{m+nx-ry}+\frac{r}{\frac{m}{x}+n-rx}$$

$$=\frac{m}{mx+nx^2-rx^3}+\frac{n}{m+nx-rx^2}+\frac{r}{\frac{m}{x}+n-rx}$$

$$=\frac{m}{mx+nx^2-rx^3}+\frac{nx}{mx+nx^2-rx^3}+\frac{rx^2}{mx+nx^2-rx^3}=\frac{m+nx+rx^2}{x(m+nx-rx^2)}$$

$$=\frac{m+nx-rx^2+2rx^2}{x(m+nx-rx^2)}=\frac{1}{x}+\frac{2rx}{m+nx-rx^2}=\frac{1}{x}+\frac{2r}{\frac{m}{x}+n-rx}$$

So

$$\frac{ma^3+nb^3-rc^3}{mb^3+nc^3-rd^3}=\frac{a^3}{b^3}+\frac{2r}{\left(m\frac{a^3}{b^3}+n-r\frac{b^3}{a^3}\right)}$$

Answer: $\frac{a^3}{b^3}+\frac{2r}{\left(m\frac{a^3}{b^3}+n-r\frac{b^3}{a^3}\right)}$