Answer on Question#41421 – Math - Integral Calculus

We have the equation of the ellipse:

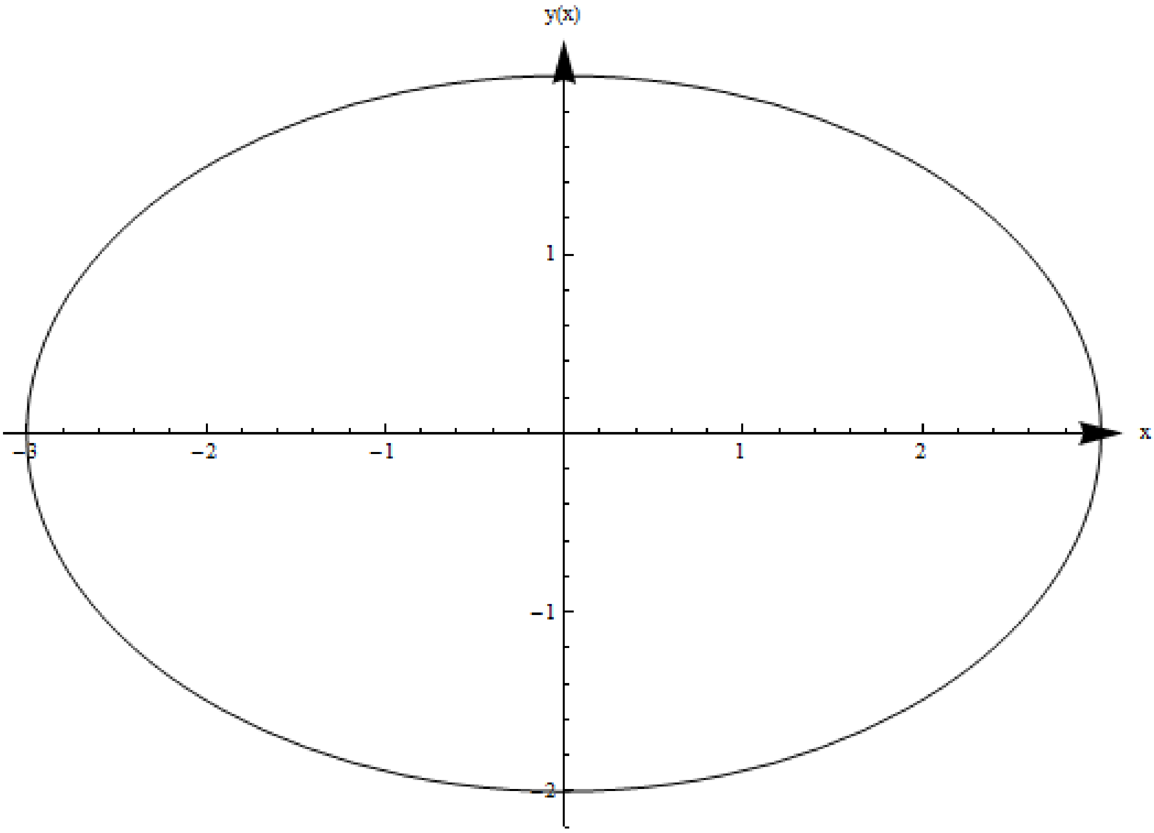
$$\frac{x^2}{9}+\frac{y^2}{4}=4 \Big| \times \frac{1}{4}$$

or

$$\frac{x^2}{4\cdot 3^2}+\frac{y^2}{4\cdot 2^2}=1$$

or

$$\frac{x^2}{6^2}+\frac{y^2}{4^2}=1$$



Let A be the area of the region D bounded by the ellipse. Since we may parameterize ∂D, with counterclockwise orientation, by $\varphi(t)=(6\cos t, 4\sin t)$,

where $t\in[0,2\pi]$. Then using Green's theorem we have

$$A=\frac{1}{2}\int_{\partial D} xdy-ydx=\frac{1}{2}\int_0^{2\pi}(-4\sin t, 6\cos t)\cdot(-6\sin t, 4\cos t)dt$$

$$=\frac{1}{2}\int_0^{2\pi}(6\cdot 4\sin^2 t+6\cdot 4\cos^2 t)dt=\frac{6\cdot 4}{2}\int_0^{2\pi}dt=12\cdot 2\pi=24\pi$$

Answer: $A=24\pi$.