Answer on Question#41419, Math, Integral Calculus

Evaluate fxy at a point (x,y) for the function f defined by f(x,y)=x (1/tan y) .
Using Schwarz's Theorem evaluate fyx at the point (x,y) .

Solution.

$$f(x,y)=\frac{x}{\tan y}$$

$$f_x(x,y)=\frac{1}{\tan y}$$

$$f_{xy}(x,y)=\left(\frac{1}{\tan y}\right)_y=-\frac{1}{\tan^2 y}\left(\frac{1}{\cos^2 y}\right)=-\frac{1}{\sin^2 y}$$

THEOREM (H. A. Schwarz). Suppose that f is a function of two variables

such that f''_{xy} and f''_{yx} both exist and are continuous at some point $(x_0; y_0)$. Then

$$f''_{xy}(x_0;\ y_0)=f''_{yx}(x_0;\ y_0)$$

Thus,

$$f_{yx}(x,y)=f_{xy}(x,y)=-\frac{1}{\sin^2 y}=-\csc y$$

Answer: $f_{yx}(x,y)=f_{xy}(x,y)=-\csc y$