Answer on Question # 41418 – Math - Calculus

Show that the equation x5+y5-16 x3 y -1=0 determines a solution fi around the point x=1 such that fi(1)=2 . Find the first derivative of the solution and its value at (1,2) .

Solution.

We have the equation

$$x^5 + y^5 - 16x^3y - 1 = 0$$

or

$$F(x,y) = 0$$

From the Implicit Function Theorem we can write the equation as

$$F\big(x, \phi(x)\big) = 0 \quad \Leftrightarrow \quad y = \phi(x)$$

Let $y = 2$ at $x = 1$ (or $\phi(1) = 2$):

$$1^5 + 2^5 - 16 \cdot 1^5 \cdot 2 - 1 = 1 + 32 - 32 - 1 = 0$$

So our equation determines the solution around the point $x = 1$.

Then find first derivative of the solution at (1,2):

$$f'(x) = -\frac{F_x'(x,y)}{F_y'(x,y)} = -\frac{5x^4 - 48x^2y}{5y^4 - 16x^3}$$

$$f'(1) = -\frac{F_x'(1,2)}{F_y'(1,2)} = -\frac{5 \cdot 1^4 - 48 \cdot 1^2 \cdot 2}{5 \cdot 2^4 - 16 \cdot 1^3} = \frac{91}{64}$$

Answer: $f'(1) = \frac{91}{64}$.