Answer on Question # 41406 – Math – Integral Calculus

Using Lagrange's multiplier method, find the extreme point of $(x, y) = xy$, subject to $x + y = 1$.

Solution.

Introduce the Lagrange's function:

$$L(x, y, \lambda) = xy + \lambda(x + y - 1);$$

If $A(x_0, y_0)$ is an extreme point, then A satisfies the following system of equations:

$$\begin{cases} \dfrac{\partial L}{\partial x} = 0 \\ \dfrac{\partial L}{\partial y} = 0 \\ \dfrac{\partial L}{\partial \lambda} = 0 \end{cases} \Rightarrow \begin{cases} y + \lambda = 0 \\ x + \lambda = 0 \\ x + y - 1 = 0 \end{cases} \Rightarrow \begin{cases} y = -\lambda \\ x = -\lambda \\ -2\lambda - 1 = 0 \end{cases} \Rightarrow \begin{cases} y = \dfrac{1}{2} \\ x = \dfrac{1}{2} \\ \lambda = -\dfrac{1}{2} \end{cases};$$

Hence, $A\left(\frac{1}{2}, \frac{1}{2}\right)$ is an extreme point.

Answer.

$A\left(\frac{1}{2}, \frac{1}{2}\right)$.