

Answer on Question #41405, Integral Calculus

Find the second Taylor polynomial of the function , f given by
 $f(x, y) = e^{(x+2y)}$ at $(-1,1)$.

Solution.

For a function that depends on two variables, x and y , the Taylor series to second order about the point (a, b) is

$$f(x, y) = f(a, b) + (x - a)f_x(a, b) + (y - b)f_y(a, b) + \frac{1}{2!}((x - a)^2 f_{xx}(a, b) + 2(x - a)(y - b)f_{xy}(a, b) + (y - b)^2 f_{yy}(a, b))$$

For our function.

$$f(a, b) = f(-1, 1) = e^{(-1+2)} = e$$

$$f_x(x, y) = e^{(x+2y)}$$

$$f_y(x, y) = 2e^{(x+2y)}$$

$$f_{xy}(x, y) = (f_x(x, y))_y = (e^{(x+2y)})_y = 2e^{(x+2y)}$$

$$f_{xx}(x, y) = (e^{(x+2y)})_x = e^{(x+2y)}$$

$$f_{yy}(x, y) = (2e^{(x+2y)})_y = 4e^{(x+2y)}$$

$$f_x(-1, 1) = e^{(-1+2)} = e$$

$$f_y(-1, 1) = 2e^{(-1+2)} = 2e$$

$$f_{xy}(-1, 1) = 2e^{(-1+2)} = 2e$$

$$f_{xx}(-1, 1) = e^{(-1+2)} = e$$

$$f_{yy}(-1, 1) = 4e^{(-1+2)} = 4e$$

Thus,

$$\begin{aligned} f(x, y) &= e + (x + 1)e + (y - 1)2e + \frac{1}{2}[(x + 1)^2 e + 2(x + 1)(y - 1)2e + (y - 1)^2 4e] = \\ &= e + e(x + 1) + 2e(y - 1) + \frac{e}{2} (x + 1)^2 + e(x^2 - 1) + 2e(y - 1)^2 \end{aligned}$$

Answer:

$$f(x, y) = e + e(x + 1) + 2e(y - 1) + \frac{e}{2} (x + 1)^2 + e(x^2 - 1) + 2e(y - 1)^2$$