Answer on Question #41403, Math, Statistics and Probability

Two floppies are selected at random without replacement from a box containing 7 good and 3 defective floppies. Let A be the event that the first floppy drawn is defective, and let B be the event that the second floppy drawn is defective.

(i) Find the conditional probabilities $P(B|A)$ and $P(B|A^c)$.

(ii) Show that $P(B) = P(B|A)P(A) + P(B|A^c)P(A^c) = P(A)$.

Solution

The probability that the first floppy drawn is defective is $P(A) = \frac{3}{10}$. The probability that the first floppy drawn is good is $P(A^c) = \frac{7}{10}$.

(i) The probability that the second floppy drawn is defective if the first floppy drawn is defective is

$$P(B|A) = \frac{2}{9},$$

because 1 defective floppy from $7 + 3 = 10$ was drawn.

The probability that the second floppy drawn is defective if the first floppy drawn is good is

$$P(B|A^c) = \frac{3}{9},$$

because 1 good floppy from 7+3=10 was drawn.

(ii) $P(B) = P(B|A)P(A) + P(B|A^c)P(A^c) = \frac{2}{9} \cdot \frac{3}{10} + \frac{3}{9} \cdot \frac{7}{10} = \frac{3(2+7)}{90} = \frac{3}{10} = P(A).$