Answer Question #41401, Integral Calculus

Find and classify the stationary points of $f(x,y) = y^2 - x^2 + 3xy$.

Solution:

$$\begin{cases} \dfrac{\partial f}{\partial x} = -2x + 3y = 0 \\ \dfrac{\partial f}{\partial y} = 2y + 3x = 0 \end{cases} \quad => \quad x = 0\,; y = 0$$

$$(0,0) - is\ a\ critical\ (stationary)\ point$$

$$\begin{cases} \dfrac{\partial^2 f}{\partial x^2} = -2 \\ \dfrac{\partial^2 f}{\partial y^2} = 2 \\ \dfrac{\partial^2 f}{\partial x \partial y} = 3 \end{cases} \quad => \quad D = \begin{pmatrix} -2 & 3 \\ 3 & 2 \end{pmatrix}$$

Determinant of matrix D is:

$$\begin{vmatrix} -2 & 3 \\ 3 & 2 \end{vmatrix} = -2 * 2 - 3 * 3 = -4 - 9 = -13 < 0$$

And matrix D don't depends of x and y, so all critical points are saddle points. It means that $(0,0)$ is a saddle point.