Answer on Question # 41380, Math, Integral Calculus

Compute the jacobian matrices using the chain rule for z=u2+v2 .
where u=2x+7 , v=3x+y+7 .

Solution:

We have function z(u,v) depends on 2 variables u and v. So the Jacobian matrix is the next:

Jacobian matrix of composition is equal to product of Jacobian matrices.

$J_{\psi\circ\varphi}(x) = J_{\psi}(\varphi(x))J_{\varphi}(x)$

$$\varphi(u, v) = u^2 + v^2$$

$$\vec{\psi}(x, y): \begin{pmatrix} u = 2x + 7 \\ v = 3x + y + 7 \end{pmatrix}$$

$$J_{\varphi} = \begin{pmatrix} \frac{\partial\varphi}{du} & \frac{\partial\varphi}{dv} \end{pmatrix} = (2u \;\; 2v) = (4x + 14 \quad 6x + 2y + 14)$$

$$J_{\psi} = \cdots \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{dy} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$$

$$\boldsymbol{J = J_{\varphi} * J_{\psi} = (4x + 14 \quad 6x + 2y + 14)\begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}}$$
$$\boldsymbol{= (24x + 6y + 70 \qquad 6x + 2y + 14)}$$