

Answer on Question #41222 – Math - Statistics and Probability

At a university, the average cost of books per student has been \$400 per student per semester. The Dean of Students believes that the costs are increasing and that the average is now greater than \$400. He surveys a sample of 40 students and finds that for the most recent semester their average cost was \$430 with a standard deviation of \$80. What is the test value for this hypothesis test?

Solution

The formulation of the null and alternative hypotheses should be

$$H_0: \mu = 400 \text{ versus } H_1: \mu > 400 .$$

The t test statistic (test value) is $T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{430 - 400}{\frac{80}{\sqrt{40}}} = 2.37$, degrees of freedom $d.f. = n - 1 = 39$.

We test at the level of significance $\alpha = 0.05$. Since H_1 is one-tailed, we set the rejection region

$$R: T \geq t_{0.05}.$$

From the t table we find that $t_{0.05}$ with $d.f. = 39$ is close to 1.684 (but it is smaller than this value). Because the observed value $t = 2.37$ is greater than 1.684, the null hypothesis is rejected at $\alpha = 0.05$.

Conclusion: there is strong evidence that the average cost of books is now greater than \$400 (with $\alpha = 0.05$).

Remark. Since $n = 40$ (large sample), we can also use normal approximation (the z test statistic

$$Z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \text{ is the same, rejection region is } R: Z \geq z_{0.05} = 1.645, \text{ conclusion is the same).}$$