

Answer on Question # 41216 – Math – Linear Algebra

Find z by the use of determinant :

$$\begin{cases} 3x - 4y + 2z + 8 = 0 \\ x + 5y - 3z + 2 = 0 \\ 5x + 3y - z + 6 = 0 \end{cases}$$

Solution.

$$\begin{cases} 3x - 4y + 2z + 8 = 0 \\ x + 5y - 3z + 2 = 0 \\ 5x + 3y - z + 6 = 0 \end{cases} \Leftrightarrow \begin{cases} 3x - 4y + 2z = -8 \\ x + 5y - 3z = -2 \\ 5x + 3y - z = -6 \end{cases} \Leftrightarrow \begin{pmatrix} 3 & -4 & 2 \\ 1 & 5 & -3 \\ 5 & 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -8 \\ -2 \\ -6 \end{pmatrix};$$

Hence:

$$z = \frac{\begin{vmatrix} 3 & -4 & -8 \\ 1 & 5 & -2 \\ 5 & 3 & -6 \end{vmatrix}}{\begin{vmatrix} 3 & -4 & 2 \\ 1 & 5 & -3 \\ 5 & 3 & -1 \end{vmatrix}} =$$

$$= \frac{3 \cdot 5 \cdot (-6) + 1 \cdot 3 \cdot (-8) + 5 \cdot (-4) \cdot (-2) - 5 \cdot 5 \cdot (-8) - 1 \cdot (-4) \cdot (-6) - 3 \cdot 3 \cdot (-2)}{3 \cdot 5 \cdot (-1) + 1 \cdot 3 \cdot 2 + 5 \cdot (-4) \cdot (-3) - 5 \cdot 5 \cdot 2 - 1 \cdot (-4) \cdot (-1) - 3 \cdot 3 \cdot (-3)}$$

$$= \frac{-90 - 24 + 40 + 200 - 24 + 18}{-15 + 6 + 60 - 50 - 4 + 27} = \frac{120}{24} = 5.$$

Answer.

$$z = 5.$$