Answer on Question#41205 – Math – Differential Calculus

Solve: $(x^3+x)^2(x^2+x)-6 = 0$

Solution.
Alternate form:
$x^8+x^7+2x^6+2x^5+x^4+x^3-6 = 0$

The degree four is the highest degree such that every polynomial equation can be solved by radicals.
So, we can use Newton method to get approximate solution of the equation.

Newton method:
If we have function **f(x)** and we know, that **a < x < b** (x – the root of this function), then:

$x \approx b - \frac{f(b)}{f'(b)}$ - if f(b) and f'(x) have same sign in given interval [a,b]

$x \approx a - \frac{f(a)}{f'(a)}$ - if f(a) and f'(x) have same sign in given interval [a,b]

For calculation of more precise value of root we can use formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

where x_n - previous value of root, x_{n+1} - next value of root.
The precision of calculation is:

$$\frac{|f(x_n)|}{m}$$

where **m** – minimal value |f'(x)| in given interval.

We have:
$f(x)= x^8+x^7+2x^6+2x^5+x^4+x^3-6$
$f'(x)= 8x^7+7x^6+12x^5+10x^4+4x^3+3x^2$
and $-2 < x_1 < -1$ (because $f(-2)=194>0$ and $f(-1)=-6<0$)
$0 < x_2 < 1$ (because $f(0)=-6<0$ and $f(1)=2>0$),
where x_1, x_2 – roots of the equation.

$$x_1 = -2 - \frac{f(-2)}{f'(-2)} \qquad x_2 = 1 - \frac{f(1)}{f'(1)}$$

Answer.
After the 1st step of approximation:

$$x_1 \approx -1.76 \qquad x_2 \approx 0.95$$

After the 2nd step of approximation:

$$x_1 = -1.76 - \frac{f(-1.76)}{f'(-1.76)} = -1.56$$

$$x_2 \approx 0.95$$

After the 3rd step of approximation:

$$x_1 = -1.56 - \frac{f(-1.56)}{f'(-1.56)} = -1.42$$

$$x_2 \approx 0.95$$

After the 4th step of approximation:

$$x_1 = -1.42 - \frac{f(-1.42)}{f'(-1.42)} = -1.35$$

$$x_2 \approx 0.95$$

After the 5th step of approximation:

$$x_1 = -1.35 - \frac{f(-1.35)}{f'(-1.35)} = -1.33$$

Precision:

$$\frac{|f(-1.33)|}{m} = \frac{0.04}{4} = 0.01$$

$$x_2 \approx 0.95$$