Answer on question #41199 – Math – Linear Algebra

We have

$$A = \begin{pmatrix} 2 & 2 & -2 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

To find the eigen values of this matrix we should find the following determinant and equate it to 0

$$|A - \lambda E| = \begin{vmatrix} 2-\lambda & 2 & -2 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = (2-\lambda)^2(3-\lambda) - 4 + 2 + 2(3-\lambda) - 4(2-\lambda) = 0$$

$$-\lambda^3 + 7\lambda^2 - 14\lambda + 8 = 0$$

$$(\lambda - 1)(\lambda - 2)(\lambda - 4) = 0$$

$$\lambda = 1; 2; 4.$$

The corresponding eigen vectors are

For $\lambda = 1$:

$$Ax - Ex = 0$$

$$\begin{pmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix} x = 0$$

$$\begin{cases} x_1 + 2x_2 - 2x_3 = 0 \\ x_1 + 2x_2 + x_3 = 0 \\ x_1 + 2x_2 + x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = -2x_2 + 2x_3 \\ -2x_2 + 2x_3 + 2x_2 + x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_3 = 0 \\ x_1 = -2x_2 \end{cases}$$

The eigen vector is $x = (-2; 1; 0)$.

For $\lambda = 2$:

$$Ax - 2Ex = 0$$

$$\begin{pmatrix} 0 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} x = 0$$

$$\begin{cases} 2x_2 - 2x_3 = 0 \\ x_1 + x_2 + x_3 = 0 \\ x_1 + 2x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = -2x_2 \\ x_3 = x_2 \\ -2x_2 + x_2 + x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = -2; \\ x_2 = 1; \\ x_3 = 1 \end{cases}$$

The eigen vector is $x = (-2; 1; 1)$.

For $\lambda = 4$:

$$Ax - 4Ex = 0$$

$$\begin{pmatrix} -2 & 2 & -2 \\ 1 & -1 & 1 \\ 1 & 2 & -2 \end{pmatrix} x = 0$$

$$\begin{cases} -2x_1 + 2x_2 - 2x_3 = 0 \\ x_1 - x_2 + x_3 = 0 \\ x_1 + 2x_2 - 2x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 1 \\ x_3 = 1 \end{cases}$$

The eigen vector is $x = (0; 1; 1)$.