**Answer on Question #41166 – Math – Calculus**

Determine whether each of the following converges or diverges:

1) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$
2) $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$
3) $\sum_{k=1}^{\infty} \frac{\sin k}{k^2}$

## Solution

1) Using the Integral Test of convergence we have:

$$\int_1^{\infty} \frac{1}{x^2+1} dx = tan^{-1}(x)|_{x=1}^{x=\infty} = \frac{\pi}{4}$$

**Therefore $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ is the convergent series.**

2) Using the Ratio Test of convergence we have:

$$\lim_{n\to\infty} \frac{\frac{2^{n+1}+5}{3^{n+1}}}{\frac{2^n+5}{3^n}} = \lim_{n\to\infty} \frac{2+\frac{5}{2^n}}{3(1+\frac{5}{2^n})} = \frac{2}{3} < 1$$

**Therefore $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$ is the convergent series.**

3) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is the convergent series, because according to the Integral test

$\int_1^{\infty} \frac{1}{x^2} dx = -\frac{1}{x}|_{x=1}^{x=\infty} = 1.$

As we can see $\frac{|\sin(n)|}{n^2} \leq \frac{1}{n^2}$ .

**Therefore series $\sum_{n=1}^{\infty} \frac{sin(n)}{n^2}$ is absolutely convergent, and in particular convergent.**