Answer on Question # 41152, Math, Integral Calculus

Question:

Evaluate the limit :

lim x tends to 0+ (ln tan 2x)/(ln tan x) .

Answer:

$$\lim_{x\to+0}\frac{\ln\tan 2x}{\ln\tan x}$$

Using l'Hôpital's rule:

$$\lim_{x\to+0}\frac{(\ln\tan 2x)'}{(\ln\tan x)'}=\lim_{x\to+0}\frac{\left(\frac{1}{\tan 2x}\frac{1}{\cos^2 2x}\,2\right)'}{\left(\frac{1}{\tan x}\frac{1}{\cos^2 x}\right)'}=2\lim_{x\to+0}\frac{\sin x\cos x}{(\sin 2x\cos 2x)}$$

$$=2\lim_{x\to+0}\frac{\sin 2x}{\sin 4x}$$

Using $\lim_{x\to+0}\frac{\sin x}{x}=1$:

$$2\lim_{x\to+0}\frac{\sin 2x}{\sin 4x}=2\frac{2x}{4x}=1$$

Answer: 1