Answer on Question # 41150 – Math - Differential Calculus

Using Jacobi's method find the complete integral of the equation 2Axz + 3B y2+ B2 C =0.

Solution.

We have the equation:

$$2Axz + 3By^2 + B^2C = 0$$

Rewrite our equation:

$$2a_1x_1x_3 + 3a_2x_2^2 + a_2^2a_3 = 0,$$

where $\{a_1, a_2, a_3\} = \{A, B, C\}$ and $\{x_1, x_2, x_3\} = \{x, y, z\}$.

It is the Hamilton-Jacobi equation in the form:

$$S(x_1, x_2, x_3, a_1, a_2, a_3) = 0$$

The sequences

$$\frac{\partial S}{\partial a_i} = b_j, b_j = \text{const},$$

determine the solutions of the equation.

So find it:

$$\frac{\partial S}{\partial A} = 2xz, \qquad \frac{\partial S}{\partial B} = 3y^2 + 2BC, \qquad \frac{\partial S}{\partial C} = B^2$$

$$\frac{\partial S}{\partial x} = 2Az, \qquad \frac{\partial S}{\partial y} = 6By, \qquad \frac{\partial S}{\partial z} = 2Ax$$

Answer:

$$\begin{cases} \dfrac{\partial S}{\partial A} = 2xz, \\ \dfrac{\partial S}{\partial B} = 3y^2 + 2BC, \\ \dfrac{\partial S}{\partial C} = B^2, \end{cases} \qquad \begin{cases} \dfrac{\partial S}{\partial x} = 2Az \\ \dfrac{\partial S}{\partial y} = 6By \\ \dfrac{\partial S}{\partial z} = 2Ax \end{cases}$$