Answer on Question #41149 – Math - Integral Calculus

We have the function

$$f(x,y,z) = x^2 + 2y + z,$$

which is integrable on $[0,2] \times [2,4] \times [4,6]$.

Find the integral:

$$I = \int_0^2 dx \int_2^4 dy \int_4^6 dz \cdot f(x,y,z) = \int_0^2 dx \int_2^4 dy \int_4^6 dz \cdot (x^2 + 2y + z) = \int_0^2 dx \int_2^4 dy \int_4^6 dz \cdot x^2 +$$
$$\int_0^2 dx \int_2^4 dy \int_4^6 dz \cdot 2y + \int_0^2 dx \int_2^4 dy \int_4^6 dz \cdot z$$

Evaluate the integrals:

$$\int_0^2 dx = 2 - 0 = 2$$

$$\int_2^4 dy = 4 - 2 = 2$$

$$\int_4^6 dz = 6 - 4 = 2$$

Then

$$I = \int_0^2 dx \cdot x^2 \cdot 2 \cdot 2 + \int_2^4 dy \cdot 2y \cdot 2 \cdot 2 + \int_4^6 dz \cdot z \cdot 2 \cdot 2 = 4 \cdot \left(\frac{x^3}{3}\bigg|_0^2 + y^2\Big|_2^4 + \frac{z^2}{2}\bigg|_4^6 \right)$$
$$= 4 \cdot \left(\frac{8}{3} + 16 - 4 + \frac{36}{2} - \frac{16}{2} \right) = 4 \cdot \left(\frac{8}{3} + 22 \right) = 4 \cdot \frac{74}{3} = \frac{296}{3}$$

Answer:

$$I = \int_0^2 dx \int_2^4 dy \int_4^6 dz \cdot (x^2 + 2y + z) = \frac{296}{3}$$