

Answer on Question # 41100 – Math - Complex Analysis

Show that $z_1(z_2+z_3)=z_1z_2+z_1z_3$ for all complex numbers z_1, z_2 and z_3 .

Solution

Let $z_1 = a_1 + ib_1, z_2 = a_2 + ib_2, z_3 = a_3 + ib_3$ are arbitrary complex numbers.

$$z_2 + z_3 = a_2 + ib_2 + a_3 + ib_3 = (a_2 + a_3) + i(b_2 + b_3).$$

Then

$$\begin{aligned} z_1(z_2 + z_3) &= (a_1 + ib_1)((a_2 + a_3) + i(b_2 + b_3)) \\ &= a_1(a_2 + a_3) + ia_1(b_2 + b_3) + ib_1(a_2 + a_3) - b_1(b_2 + b_3). \end{aligned}$$

Let's simplify this expression

$$z_1(z_2 + z_3) = (a_1a_2 + ia_1b_2 + ib_1a_2 - b_1b_2) + (a_1a_3 + ia_1b_3 + ib_1a_3 - b_1b_3).$$

But

$$(a_1a_2 + ia_1b_2 + ib_1a_2 - b_1b_2) = (a_1 + ib_1)(a_2 + ib_2) = z_1z_2,$$

and

$$(a_1a_3 + ia_1b_3 + ib_1a_3 - b_1b_3) = (a_1 + ib_1)(a_3 + ib_3) = z_1z_3.$$

So

$$z_1(z_2 + z_3) = z_1z_2 + z_1z_3.$$