

Answer on Question #41084 – Math - Differential Calculus | Equations

First, function $h(x) = \sqrt{9 - x^2} = f(g(x))$ is a composite function, where $f(x) = \sqrt{x}$, $g(x) = 9 - x^2$.

We know derivatives of the following functions:

$$(f(x))' = (\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$(g(x))' = (9 - x^2)' = |\text{derivative of a difference}| = (9)' - (x^2)' = 0 - 2x = -2x$$

Derivative of a composite function is calculated by the formula

$$(\sqrt{9 - x^2})' = (h(x))' = (f(g(x)))' = f'(g(x)) \times g'(x) = \frac{1}{2\sqrt{9 - x^2}} \times (-2x) = -\frac{x}{\sqrt{9 - x^2}}$$

Further, calculate

$$\begin{aligned} \left(\frac{2}{x}\sqrt{9 - x^2}\right)' &= |\text{derivative of product}| = \left(\frac{2}{x}\right)' \left(\sqrt{9 - x^2}\right) + \left(\frac{2}{x}\right) \left(\sqrt{9 - x^2}\right)' = \\ &= 2\left(\frac{1}{x}\right)\sqrt{9 - x^2} + \frac{2}{x}\left(\sqrt{9 - x^2}\right)' = 2 \times \frac{(-1)}{x^2} \times \sqrt{9 - x^2} + \frac{2}{x}\left(\sqrt{9 - x^2}\right)' = \\ &|\text{use the previous result}| \\ &= -\frac{2\sqrt{9 - x^2}}{x^2} + \frac{2}{x} \times \frac{(-x)}{\sqrt{9 - x^2}} = -\frac{2\sqrt{9 - x^2}}{x^2} - \frac{2}{\sqrt{9 - x^2}} = \frac{-2}{x^2\sqrt{9 - x^2}}(9 - x^2 + x^2) = -\frac{18}{x^2\sqrt{9 - x^2}}. \end{aligned}$$

Domain:

$$-3 < x < 0 \text{ or } 0 < x < 3$$