

Answer on Question # 40996 – Math – Linear algebra

Solve the set of linear equations by the matrix method : $a+3b+2c=3$, $2a-b-3c=-8$, $5a+2b+c=9$. Solve for c

Solution:

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} x \\ x \\ 2 \\ x \\ 3 \end{pmatrix}$$

$$\mathbf{A} \cdot \mathbf{X} = \mathbf{B}$$

so

$$\mathbf{X} = \mathbf{A}^{-1} \cdot \mathbf{B}$$

Find the determinant of a matrix $\mathbf{A}$

$$\det \mathbf{A} = -28$$

To find the inverse matrix to calculate the cofactors of the elements of the matrix $\mathbf{A}$

$$C_{1,1} = (-1)^{1+1} \begin{vmatrix} -1 & -3 \\ 2 & 1 \end{vmatrix} = 5$$

$$C_{1,2} = (-1)^{1+2} \begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} = -17$$

$$C_{1,3} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = 9$$

$$C_{2,1} = (-1)^{2+1} \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = 1$$

$$C_{2,2} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = -9$$

$$C_{2,3} = (-1)^{2+3} \begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} = 13$$

$$C_{3,1} = (-1)^{3+1} \begin{vmatrix} 3 & 2 \\ -1 & -3 \end{vmatrix} = -7$$

$$C_{3,2} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = 7$$

$$C_{3,3} = (-1)^{3+3} \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -7$$

$$\mathbf{C} = \begin{pmatrix} 5 & -17 & 9 \\ 1 & -9 & 13 \\ -7 & 7 & -7 \end{pmatrix}$$

$$\mathbf{C}^T = \begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}$$

Let us find the inverse of a matrix

$$\mathbf{A}^{-1} = \frac{\mathbf{C}^T}{\det \mathbf{A}} = \begin{pmatrix} -5/28 & -1/28 & 1/4 \\ 17/28 & 9/28 & -1/4 \\ -9/28 & -13/28 & 1/4 \end{pmatrix}$$

And a solution

$$\mathbf{X} = \mathbf{A}^{-1} \cdot \mathbf{B} = \begin{pmatrix} -5/28 & -1/28 & 1/4 \\ 17/28 & 9/28 & -1/4 \\ -9/28 & -13/28 & 1/4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$$

Answer: a = 2 , b = -3 , c = 5 .