Answer on Question #40883, Math, Statistics and Probability

Dottie's Tax Service specializes in federal tax returns for professional clients, such as physicians, dentists, accountants, and lawyers. A recent audit by the IRS of the returns she prepared indicated that an error was made on 14 percent of the returns she prepared last year. Assuming this rate continues into this year and she prepares 54 returns. What is the probability that she makes errors on:

(a) More than 8 returns? (Round z-score computation to 2 decimal places and your final answer to 4 decimal places.)

(b) At least 8 returns? (Round z-score computation to 2 decimal places and your final answer to 4 decimal places.)

(c) Exactly 8 returns? (Round z-score computation to 2 decimal places and your final answer to 4 decimal places.)

Solution

We can use normal approximation. Given:

$$p = 14\% = 0.14 \text{ and } q = 100\% - 14\% = 86\% = 0.86.$$

Where

$$\mu = np = 54 \cdot 0.14 = 7.56.$$

$$\sigma^2 = npq = 54 \cdot 0.14 \cdot 0.86 = 6.5.$$

$$z = \frac{x-\mu}{\sigma}.$$

(a) More than 8 returns?

Since normal distribution is continuous we should apply the continuity correction.

The point 8 is extended between 7.5 and 8.5. So "More than 8 returns" means 8.5 should be used.

$$z = \frac{x-\mu}{\sigma} = \frac{8.5-7.56}{\sqrt{6.5}} = 0.37.$$

$$P(More\ than\ 8\ returns) = \mathrm{P}(\mathrm{z} > 0.37) = 0.3557.$$

(b) At least 8 returns?

It includes 7.5 and up.

$$z = \frac{x-\mu}{\sigma} = \frac{7.5-7.56}{\sqrt{6.5}} = -0.02.$$

$$P(\text{At least 8 returns}) = \mathrm{P}(\mathrm{z} > -0.02) = 1 - \mathrm{P}(\mathrm{z} < -0.02) = 1 - 0.4920 = 0.5080.$$

(c) Exactly 8 returns?

$$P(\text{Exactly 8 returns}) = P(\text{At least 8 returns}) - P(More\ than\ 8\ returns) = 0.5080 - 0.3557 = 0.1523.$$