

Answer on Question #40855 – Math - Linear Algebra

Question:

Solve the set of linear equations by the matrix method : $a+3b+2c=3$, $2a-b-3c= -8$, $5a+2b+c=9$. Solve for c.

Solution:

The matrix of this system is $A=\begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -1 \\ 5 & 2 & 1 \end{pmatrix}$, the column of free terms is $b = \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix}$.

In matrix form this equation has presentation $Ax=b$, where $x = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

Let premultiply both sides of the equation by A^{-1} , the inverse of A.

Since $A^{-1}Ax = Ix = x$, we know the following.

$$x = A^{-1}b$$

So let's find the inverse of A. Compute the algebraic addition to the elements of the matrix A

$$M_{1,1} = (-1)^{1+1} \begin{vmatrix} -1 & -3 \\ 2 & 1 \end{vmatrix} = 5$$

$$M_{1,2} = (-1)^{1+2} \begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} = -17$$

$$M_{1,3} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = 9$$

$$M_{2,1} = (-1)^{2+1} \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = 1$$

$$M_{2,2} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = -9$$

$$M_{2,3} = (-1)^{2+3} \begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} = 13$$

$$M_{3,1} = (-1)^{3+1} \begin{vmatrix} 3 & 2 \\ -1 & -3 \end{vmatrix} = -7$$

$$M_{3,2} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = 7$$

$$M_{3,3} = (-1)^{3+3} \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -7$$

$$C^* = \begin{pmatrix} 5 & -17 & 9 \\ 1 & -9 & 13 \\ -7 & 7 & -7 \end{pmatrix}$$

$$C^{*T} = \begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}$$

So, an inverse of A is

$$A^{-1} = \frac{C^{*T}}{\det A} = \begin{pmatrix} -\frac{5}{28} & -\frac{1}{28} & \frac{1}{4} \\ \frac{17}{28} & \frac{9}{28} & -\frac{1}{4} \\ -\frac{9}{28} & -\frac{13}{28} & \frac{1}{4} \end{pmatrix}$$

Thus our solution is

$$x = A^{-1}b = \begin{pmatrix} -\frac{5}{28} & -\frac{1}{28} & \frac{1}{4} \\ \frac{17}{28} & \frac{9}{28} & -\frac{1}{4} \\ -\frac{9}{28} & -\frac{13}{28} & \frac{1}{4} \end{pmatrix} * \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$$

Answer: a=2,b=-3, c=5.