Answer on Question#40793 - Math - Linear Algebra

Task:

Let a quadratic form have the expression x2+y2+2z2+2xy+3xz with respect to the standard basis B1 = f(1;0;0); (0;1;0); (0;0;1)g. Find its expression with respect to the basis B2 = f(1;1;1); (0;1;0); (0;1;1)g

Solution:

Denote this quadratic form by Q= $x^2 + y^2 + 2z^2 + 2xy + 3xz$;

The matrix of this quadratic form with respect to the standard basis B1 = {f(1;0;0); (0;1;0); (0;0;1)} is A = $\begin{pmatrix} 1 & 1 & 3/2 \\ 1 & 1 & 0 \\ 3/2 & 0 & 2 \end{pmatrix}$;

This means that A is a symmetric n×n matrix such that $Q = x^T Ax$;

C= $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ – is a transformation matrix from B1 to B2;

A1= $C^T AC$ - matrix of this quadratic form with respect to the basis B2;

Let find A1; A1= $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{1} & \frac{3}{2} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \frac{3}{2} & \mathbf{0} & \mathbf{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{7}{2} & 2 & \frac{7}{2} \\ 1 & 1 & 0 \\ \frac{5}{2} & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} =$

$\begin{pmatrix} 9 & 2 & 11/2 \\ 2 & 1 & 1 \\ 11/2 & 1 & 3 \end{pmatrix}$;

So the expression of Q with respect to the basis B2 = {(1;1;1); (0;1;0); (0;1;1)} is Q1 = $x^T A1x$ = $(x\ y\ z) \begin{pmatrix} 9 & 2 & \frac{11}{2} \\ 2 & 1 & 1 \\ \frac{11}{2} & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} =$

$9x^2 + y^2 + 3z^2 + 4xy + 11xz + 2yz$;

Answer:

The expression of Q with respect to the basis B2 = {f(1;1;1); (0;1;0); (0;1;1)} is Q1= $9x^2 + y^2 + 3z^2 + 4xy + 11xz + 2yz$;