Answer on question #40787 – Math – Linear Algebra

Let f:C3 to C be defined as f(z)=(z1-z2)-i(2z1+z2+z3), where z=(z1,z2,z3) belongs to C3.

Find aw belongs to C3 such that f(z)=<z,w>, where <,> is the standard inner product on C3.

Answer:

The standard inner product defines as

$$< z; w >= z_1\overline{w_1} + z_2\overline{w_2} + z_3\overline{w_3},$$

where $z = (z_1, z_2, z_3), w = (w_1, w_2, w_3)$.

So we get

$$\mathrm{f(z)} =< \mathrm{z, w} \geq z_1\overline{w_1} + z_2\overline{w_2} + z_3\overline{w_3} = (\mathrm{z_1 - z_2}) - \mathrm{i}(2\mathrm{z_1} + \mathrm{z_2} + \mathrm{z_3}) =$$

$$= z_1(1 - 2i) + z_2(-1 - i) + z_3(-i),$$

Therefore we obtain

$$\overline{w_1} = 1 - 2i \;\Rightarrow\; w_1 = 1 + 2i;$$

$$\overline{w_2} = -1 - i \;\Rightarrow\; w_2 = -1 + i;$$

$$\overline{w_3} = -i \;\Rightarrow\; w_3 = i.$$

Answer: $w = (1 + 2i; -1 + i; i)$.