Answer on Question #40758 – Math – Vector Calculus

Explain why it is not possible for two of a vector's direction angles to be less than 45.

Solution.

As it is known, there is a theorem for vector's direction angles, which states that $\cos^2\alpha_x + \cos^2\alpha_y + \cos^2\alpha_z = 1$.

This is statement can be easily derived, if one substitutes $\frac{a_x}{|\vec{a}|}$ for α_x and so on for other angles:

$$\cos^2\alpha_x + \cos^2\alpha_y + \cos^2\alpha_z = \frac{a_x^2}{|\vec{a}|^2} + \frac{a_y^2}{|\vec{a}|^2} + \frac{a_z^2}{|\vec{a}|^2} = \frac{a_x^2 + a_y^2 + a_z^2}{|\vec{a}|^2} = \frac{|\vec{a}|^2}{|\vec{a}|^2} \equiv 1.$$

If two of a vector's direction angles (for e.g., α_x and α_y) are less than 45^0, then

$$\cos^2\alpha_x + \cos^2\alpha_y + \cos^2\alpha_z > \cos^2 45^0 + \cos^2 45^0 + \cos^2\alpha_z = \frac{1}{2} + \frac{1}{2} + \cos^2\alpha_z \geq 1.$$

We obtained that $\cos^2\alpha_x + \cos^2\alpha_y + \cos^2\alpha_z > 1$, and this inequality contradicts with the initial theorem.

Answer: the statement is correct.