Answer on Question #40754 – Math - Vector Calculus

A plane sheet of material is bound by the curve y = x^2 from x = 0 to x = 1 the x-axis and the line x = 1. If the mass per unit area (density) of the sheet is xy find the mass of the sheet.

Solution:

It is known that the mass of the plane sheet can be evaluated with the formula

$m = \iint\limits_D \rho(x, y)dxdy$, where ρ is the density of the sheet

So, $m = \iint\limits_D xydxdy = \int\limits_0^1 xdx \int\limits_0^{x^2} ydy = \int\limits_0^1 x\frac{y^2}{2}\Big|_0^{x^2} dx = \int\limits_0^1 \frac{x^5}{2}dx = \frac{x^6}{12}\Big|_0^1 = \frac{1}{12}$

Answer:

The mass of this plane sheet is $\frac{1}{12}$.