Answer on Question #40704 – Math – Calculus

calculate:

1) lim 1-cos x/x+x^2   2) lim x-2x^2/3x^2+5x   3) lim (x sin 1/x)
   $x \to 0$ $x \to \infty$ $x \to \infty$

4) Show that the sequence 1/2, 2/3,3/4,4/5,... is strictly increasing sequence.

Solution:

1) $\lim_{x \to 0} \frac{1 - \cos x}{x + x^2}$

If we substitute $x = 0$ we get the form $\frac{0}{0}$.This means we can apply L'Hospital's rule. So we can take the derivative of numerator and the denomenator without changing the limits:

$$\lim_{x \to 0} \frac{1 - \cos x}{x + x^2} = \lim_{x \to 0} \frac{\sin x}{1 + 2x} = \frac{0}{1} = 0$$

2) $\lim_{x \to \infty} \frac{x - 2x^2}{3x^2 + 5x}$

We can divide numerator and the denomenator on x^2:

$$\lim_{x \to \infty} \frac{x - 2x^2}{3x^2 + 5x} = \lim_{x \to \infty} \frac{\frac{x}{x^2} - \frac{2x^2}{x^2}}{\frac{3x^2}{x^2} + \frac{5x}{x^2}} = \lim_{x \to \infty} \frac{\frac{1}{x} - 2}{3 + \frac{5}{x}} = = -\frac{2}{3}$$

3) $\lim_{x \to \infty} x \sin \frac{1}{x}$

We can apply the La'Hospital rule. For that first of all convert the equation to form such that after applying limit directly we get $\frac{0}{0}$ or $\frac{\text{infinity}}{\text{infinity}}$ form. Then differentiate both the numerator and the denominator and then apply the limit thus

$$f(x) = x \cdot \sin\left(\frac{1}{x}\right) \text{ convert to } \frac{f(x)}{g(x)} \text{ form i.e.}$$

$$\frac{f(x)}{g(x)} = \sin \frac{\left(\frac{1}{x}\right)}{\frac{1}{x}}$$

which is now in the form of $\frac{0}{0}$,then according to La'Hospital rule

$$\frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$$

Thus,

$$\frac{f'(x)}{g'(x)} = \frac{-\frac{1}{x^2 \cos \frac{1}{x}}}{-1/x^2} = \cos \frac{1}{x}$$

Now we can apply the limit to this derivative which gives

$$\cos\left(\frac{1}{\infty}\right) = \cos(0) = 1$$

$$\lim_{x\to\infty} x \sin\frac{1}{x} = 1$$

4) We have the sequence $\left(\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\right)$:

$$a_n = \frac{n}{n+1}$$

Each term is greater than the last:

$$a_{n+1} - a_n = \frac{n+1}{n+2} - \frac{n}{n+1} = \frac{n^2 + 2n + 1 - n^2 - 2n}{(n+2)(n+1)} = \frac{1}{(n+2)(n+1)} > 0 \Longrightarrow$$

The inequality is satisfied for any value of n. (n is a positive integer). Hence, the sequence is strictly monotonically increasing.