

Answer on Question #40702 – Math - Calculus

Determine whether each of the following converges or diverges:

1) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

2) $\sum_{n=0}^{\infty} \frac{2^{n+5}}{3^n}$

3) $\sum_{k=1}^{\infty} \frac{\sin k}{k^2}$

Solution

1) Using the Integral Test of convergence we have:

$$\int_1^{\infty} \frac{1}{x^2+1} dx = \tan^{-1}(x) \Big|_{x=1}^{x=\infty} = \frac{\pi}{4}$$

Therefore $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ is the convergent series.

2) Using the Ratio Test of convergence we have:

$$\lim_{n \rightarrow \infty} \frac{\frac{2^{n+1}+5}{3^{n+1}}}{\frac{2^n+5}{3^n}} = \lim_{n \rightarrow \infty} \frac{2 + \frac{5}{2^n}}{3(1 + \frac{5}{2^n})} = \frac{2}{3} < 1$$

Therefore $\sum_{n=0}^{\infty} \frac{2^{n+5}}{3^n}$ is the convergent series.

3) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is the convergent series, because according to the Integral test

$$\int_1^{\infty} \frac{1}{x^2} dx = -\frac{1}{x} \Big|_{x=1}^{x=\infty} = 1.$$

$$\text{As we can see } \frac{|\sin(n)|}{n^2} \leq \frac{1}{n^2}.$$

Therefore series $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$ is absolutely convergent, and in particular convergent.