

Answer on Question #40609 – Math – Calculus

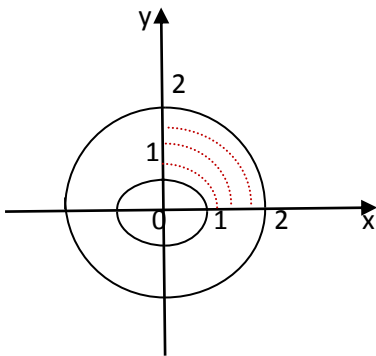
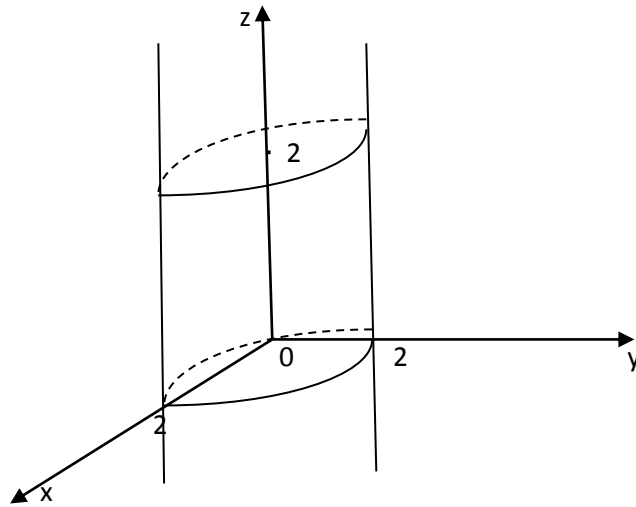
Given the triple integral $\iiint_S (x^2 + y^2)^{1/2} dx dy dz$

$$S = \{ (x, y, z) : 1 \leq x^2 + y^2 \leq 4, x \geq 0, y \geq 0, 0 \leq z \leq 2 \}$$

Sketch the solid S and evaluate the integral.

Solution:

Let sketch the solid S :



Let evaluate the integral $\iiint_S (x^2 + y^2)^{1/2} dx dy dz$

$$\text{Let } x = \rho \cos \varphi$$

$$y = \rho \sin \varphi,$$

$$dx dy = \rho d\varphi d\rho, \quad x^2 + y^2 = \rho^2.$$

Let substitute

$$\int_0^{\frac{\pi}{2}} d\varphi \int_1^2 \rho d\rho \int_0^2 \sqrt{\rho^2} dz = \frac{\pi}{2} \int_1^2 \rho^2 d\rho \cdot 2 = \pi \cdot \frac{\rho^3}{3} \Big|_1^2 = \frac{8\pi}{3} - \frac{\pi}{3} = \frac{7\pi}{3}.$$

Answer: $\frac{7\pi}{3}$