Answer on Question #40490 – Math – Differential Calculus

Solve the IVP $x^2y' = \cos x - 2xy, y(\pi) = 0, x > 0$.

Solution:

$x^2y' = cosx - 2xy$, whence

$y' + \frac{2y}{x} = \frac{cosx}{x^2}$,

Let $y = uv, u = u(x), v = v(x), y' = u'v + v'u$.

Let substitute $u'v + v'u + \frac{2uv}{x} = \frac{cosx}{x^2}, u'v + u(v' + \frac{2v}{x}) = \frac{cosx}{x^2}$, than

1) $v' + \frac{2v}{x} = 0$,

and , 2) $u'v = \frac{cosx}{x^2}$.

Let solve 1):

$\frac{dv}{dx} = -\frac{2v}{x}, \frac{dv}{v} = -\frac{2dx}{x}$,

$lnv = lnx^{-2}, v = \frac{1}{x^2}$.

Let solve 2):

$\frac{u'}{x^2} = \frac{cosx}{x^2}, u' = cosx, du = cosxdx$, whence

$u = sinx + c$, where c be some arbitrary const.

So, $y = uv = \frac{c+sinx}{x^2}$.

Let solve IVP $y(\pi) = 0, x > 0$:

$0 = \frac{c+sin\pi}{\pi^2}, c = -sin\pi = 0$.

$y = \frac{sinx}{x^2}$.

Answer: $y = \frac{sinx}{x^2}$.