Answer on Question #40487 – Math - Differential Calculus

We have

$$\varphi(x^2 + y^2 - z^2, x + y + z) = 0$$

or the equations

$$x^2 + y^2 - z^2 = c_1,$$

$$x + y + z = c_2.$$

By differentiating we get

$$xdx + ydy - zdz = 0,$$

$$dx + dy + dz = 0$$

or the equations of the form

$$xP + yQ - zR = 0,$$

$$P + Q + R = 0.$$

Our differential equation in the standard form

$$Pp + Qq = R$$

or

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

Each fraction is equal to

$$\frac{P_1 dx + Q_1 dy + R_1 dz}{P_1 P + Q_1 Q + R_1 R}$$

Multipliers may be chosen such that the numerator $P_1 dx + Q_1 dy + R_1 dz$ is an exact differential of the denominator $P_1 P + Q_1 Q + R_1 R$.

Let

$$P = z + y, Q = x + z, R = y - x$$

Then

$$d\big(P_1(z + y) + Q_1(x + z) + R_1(y - x)\big) = P_1 dx + Q_1 dy + R_1 dz$$

So we have the **first-order** differential equation

$$(z + y)p + (x + z)q = y - x.$$