

Answer on Question #40475 – Math – Differential Calculus | Equations

Find the directional derivatives of $f(x,y,z)=x^2+2y^2+3z^2$ at $P(1,1,1)$ in the direction of $a=i+j+k$

Solution:

$$f(x, y, z) = x^2 + 2y^2 + 3z^2.$$

Let find partial derivatives of function $f(x,y,z)$ at point P:

$$\frac{\partial f}{\partial x}(P) = 2x|_P = 2,$$

$$\frac{\partial f}{\partial y}(P) = 4y|_P = 4,$$

$$\frac{\partial f}{\partial z}(P) = 6z|_P = 6.$$

Than, we have $\bar{a} = 1\bar{i} + 1\bar{j} + 1\bar{k}$,

whence

$$|\bar{a}| = \sqrt{1 + 1 + 1} = \sqrt{3}$$

and

$$\frac{\partial f}{\partial \bar{a}}|_P = 2 \cdot \frac{1}{\sqrt{3}} + 4 \cdot \frac{1}{\sqrt{3}} + 6 \cdot \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}(2 + 4 + 6) = \frac{12}{\sqrt{3}} = 4\sqrt{3}.$$

Answer: $\frac{\partial f}{\partial \bar{a}}|_P = 4\sqrt{3}$.